2013 problemleri

1. m ve n pozitif tamsayılar olmak üzere; $m^2 + n^2 < 2013$ ise $100m + n$ kaçtır?

2. p 2013 den küçük en büyük asal sayı olmak üzere; $N = 20 + p^{p^{p+1}-13}$ sayısı da asal olduğuna göre N in 10^4 ile bölümünden elde edilen kalan kaçtır?

3. Aşağıdaki kesrin payı ve paydası 70 karakterden oluşmaktadır.

$$r = \frac{loooloolloolloolollllolooll ollolllloollloloolooololooolololoooooollllol}{loloolooloollollolloooooloooloololloolllooollololoooollllooololllolooool}.$$

   $o = 2013$ ve $l = \frac{1}{50}$ ise $\lceil roll \rceil = ?$

4. Bir okuldaki kız öğrencilerin sayısının erkek öğrencilerin sayısına oranı 1 : 3 olmak üzere;

   - Her erkek öğrencinin en fazla 2013 tane kız arkadaşı vardır.
   - Her kız öğrencinin en az n tane erkek arkadaşı vardır.
   - Arkadaşlıklar karşılıklıdır

   ise n in en büyük değeri kaçtır?

5. 1 den 2013 e kadar olan tamsayılar 2013×2013 lük bir ızgaranın her birim karesine yazılıp aşağıdaki işlemler yapılıyor.

   - Bir satır seç ve satırdaki her sayıdan 1 çıkart.
   - Bir kolon seç ve kolondaki her sayıya 1 ekle.

   Bu şekilde devam ederek bütün sayıların 2013 ile bölünmesini sağlayacak şekilde başlangıçta kaç farklı diziliş olur?

6. Determine all triples (x, y, z) of positive integers for which the number $\sqrt{\frac{2013}{x+y}} + \sqrt{\frac{2013}{y+z}} + \sqrt{\frac{2013}{z+x}}$ is an integer

7. Find the remainder when $2 + 4 + \cdots + 2014$ is divided by $1 + 3 + \cdots + 2013$

8. Which number is bigger $\sqrt[2012]{2012!}$ or $\sqrt[2013]{2013!}$

9. How many positive integers which are less or equal with 2013 such that 3 or 5 divide the number

10. The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!},$$

   where $a_1 \geq a_2 \geq \cdots \geq a_m$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

11. Let $x, y, z \geq 0$,such that $3x + 11y + 61z = 2013$,prove that
$$\frac{4}{61}(xy + zx + yz) \leq 2013.$$

12. Let $x, y, z \geq 0$,such that $3xy + 11yz + 61zx = 2013$,prove that
$$3z^{-1} + 11x^{-1} + 61y^{-1} \geq 3\sqrt{3}.$$

13. How many positive three-digit integers $\underline{abc}$ can represent a valid date in 2013, where either a corresponds to a month and $\underline{bc}$ corresponds to the day in that month, or $\underline{ab}$ corresponds to a month and c corresponds to the day? For example, 202 is a valid representation for February 2nd, and 121 could represent either January 21st or December 1st.

14. A board has 2, 4, and 6 written on it. A person repeatedly selects (not necessarily distinct) values for x, y, and z from the board, and writes down $xyz + xy + yz + zx + x + y + z$ if and only if that number is not yet on the board and is also less than or equal to 2013. This person repeats this process until no more numbers can be written. How many numbers will be written at the end of the process?

15. Let $f_1(n)$ be the number of divisors that n has, and define $f_k(n) = f_1(f_{k-1}(n))$. Compute the smallest integer k such that $f_k(2013^{2013}) = 2$

16. Consider a sequence given by $a_n = a_{n-1} + 3a_{n-2} + a_{n-3}$, where $a_0 = a_1 = a_2 = 1$. What is the remainder of a_{2013} divided by 7?

17. Compute the number of positive integers b where $b \leq 2013$, $b \neq 17$, and $b \neq 18$, such that there exists some positive integer N such that $\frac{N}{17}$ is a perfect 17th power, $\frac{N}{18}$ is a perfect 18th power, and $\frac{N}{b}$ is a perfect bth power.

18. Determine, with proof, the least positive integer n for which there exist n distinct positive integers $x_1, x_2, \cdots, x_n$ such that
$$\left(1 - \frac{1}{x_1}\right)\left(1 - \frac{1}{x_2}\right)\cdots\left(1 - \frac{1}{x_n}\right) = \frac{15}{2013}$$

19. Given that $x \leq y$, $xy = 2013(x + y)$, find the number of positive integer solutions.

20. Let
$$P(x) = x^{2013} + 4x^{2012} + 9x^{2011} + 16x^{2010} + \cdots + 4052169x + 4056196 = \sum_{j=1}^{2014} j^2 x^{2014-j}.$$
If $a_1, a_2, \cdots a_{2013}$ are its roots, then compute the remainder when
$a_1^{997} + a_2^{997} + \cdots + a_{2013}^{997}$ is divided by 997